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Radiation Physics Group

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PRESSURE DEPENDENCE OF THE PINCHED ELECTRON BEAM MODE

by J. Creedon

SUMMARY

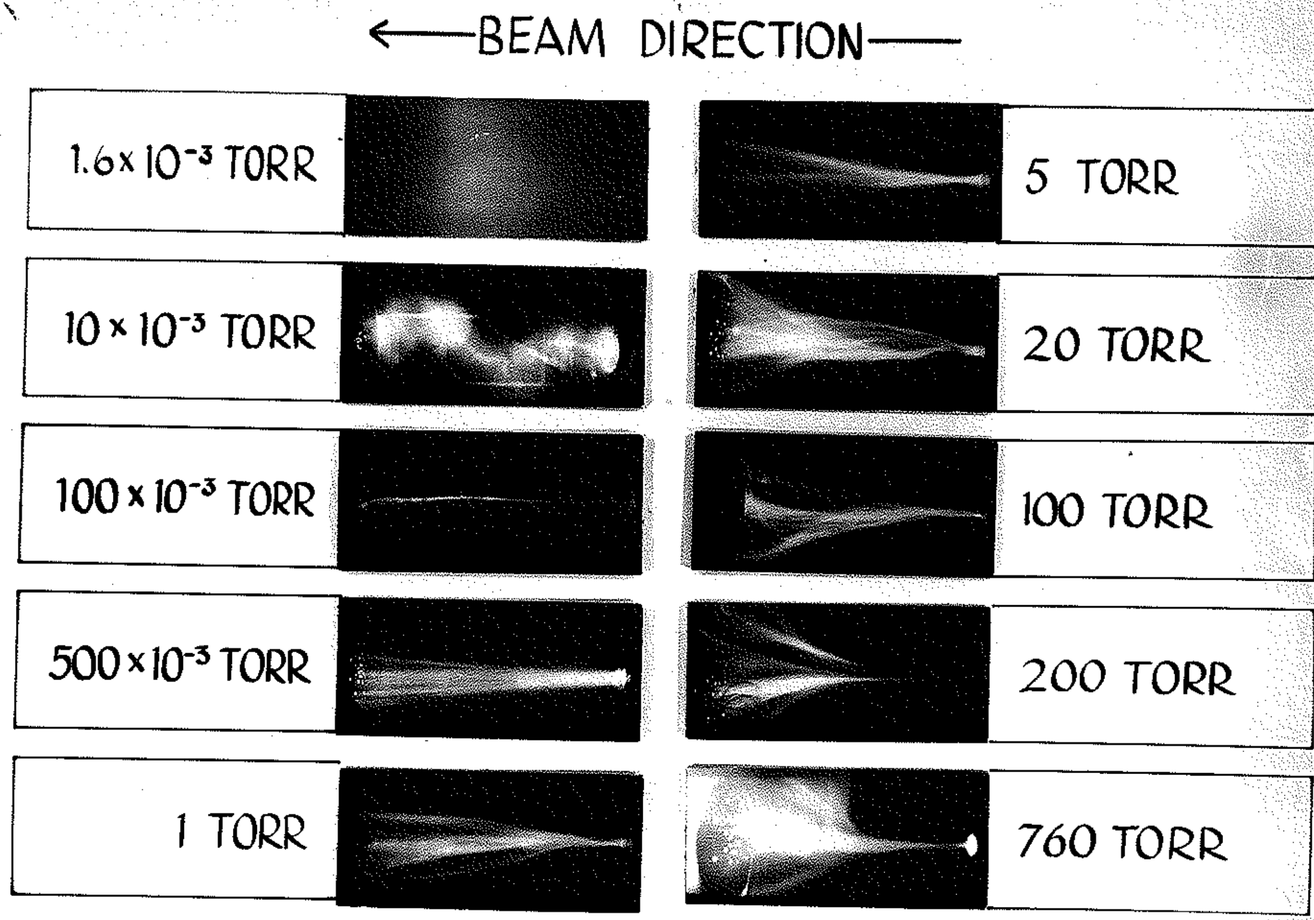
The time required for electrical breakdown in the background gas is shown to have the right order of magnitude to explain the pressure dependence of the pinched electron beam mode. Predictions made from the breakdown theory are shown to be in qualitative agreement with the available experimental data.

I. INTRODUCTION

The high current electron beams (10^4 - 10^5 amperes), available from the Physics International pulsed power systems, require some form of space charge neutralization if they are to be transported efficiently. One common way of neutralizating the space charge is to introduce a background gas in the beam drift chamber. This gas is ionized by the electron beam and (after the secondary electrons escape) provides the necessary positive charge for neutralization.

The characteristics of the electron beam depend critically on the pressure of this background gas. Figure 1 (Ref. 1) shows pictures of the electron beam taken at various gas pressures.

One of the most striking aspects of the electron beam shown in Figure 1 is a sharp change from a pinched to an unpinched mode which occurs with increasing pressure in the 0.1 to 0.35 mm range. There is a considerable amount of evidence which indicates that the path of the return current determines the nature of the electron beam. Specifically, if the return current flows outside the electron beam (in the gas or on the chamber walls) then there will be an inward



ELECTRON BEAM-GAS INTERACTION
AS A FUNCTION OF PRESSURE

FIGURE 1

magnetic force and (if the space charge is neutralized) there will be a pinch. (See Ref. 2). If the return current flows back in the gas in the same region and with the same spatial distribution as the beam, then the net current is zero and there is no magnetic force and therefore no pinch.

Figure 2 (Ref. 3) shows two pictures of the electron beam both taken at a pressure of 0.1 mm which should have given a pinched beam. The only difference between the experimental setup is that in Figure 2a the calorimeter is not connected to the drift chamber walls and in Figure 2b the calorimeter is connected. There is an obvious explanation for the fact that the beam in Figure 2a is not pinched while the beam of Figure 2b does pinch. Stopping the electron beam in a calorimeter which is unconnected to the chamber walls (and located closer to the anode than to the chamber walls) forces the return currents to flow back down the beam in the gas and destroys the pinch. When the calorimeter is connected to the chamber walls the return currents flow on the walls and the pinch is restored.

One explanation for the relation between the gas pressure and the pinching behavior of the beam is that the electrical conductivity of the gas increases sharply with increasing pressure (for some pressure range) and therefore the beam changes from a pinched to an unpinched mode as the pressure is increased. A preliminary analysis of the problem indicates that the steady state conductivity of air decreases with increasing pressure in the 0.1 to 0.5 mm range. The problem of gas conductivity for these nanosecond pulses is not simple and one cannot at this point rule out the possibility of gas conductivity as an explanation for the pinching behaviour of the beam. Work is continuing in this area.

There is a second possible explanation for the pressure dependence of the pinched mode. When the electron beam first enters the gas the degrees of ionization and the conductivity are quite low. If the time required for the gas to become highly ionized and breakdown electrically is an appreciable fraction of the beam rise time, then the return current will flow on the chamber wall before breakdown; and this current will remain on the wall after breakdown giving a

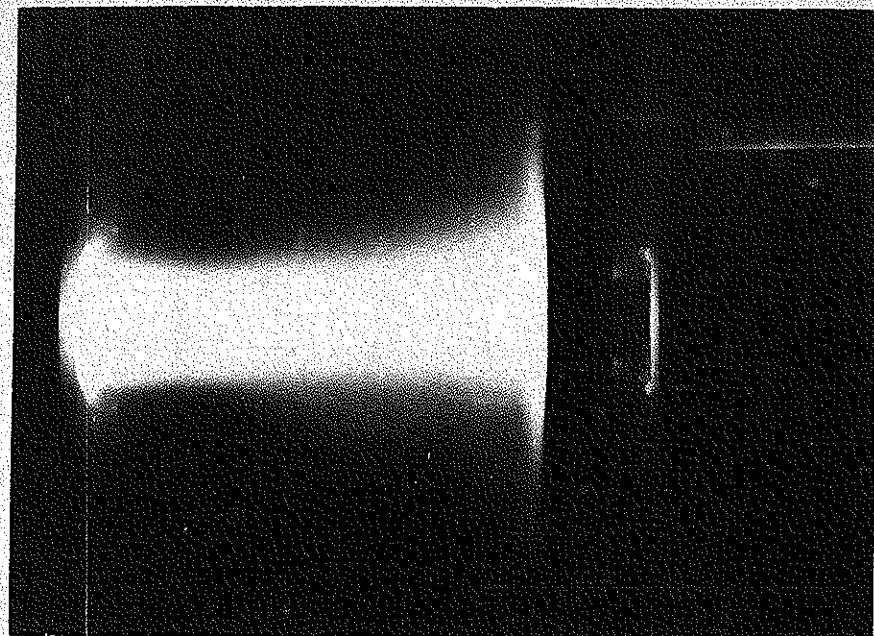


FIGURE 2a. Beam in air at .1 mm into floating calorimeter

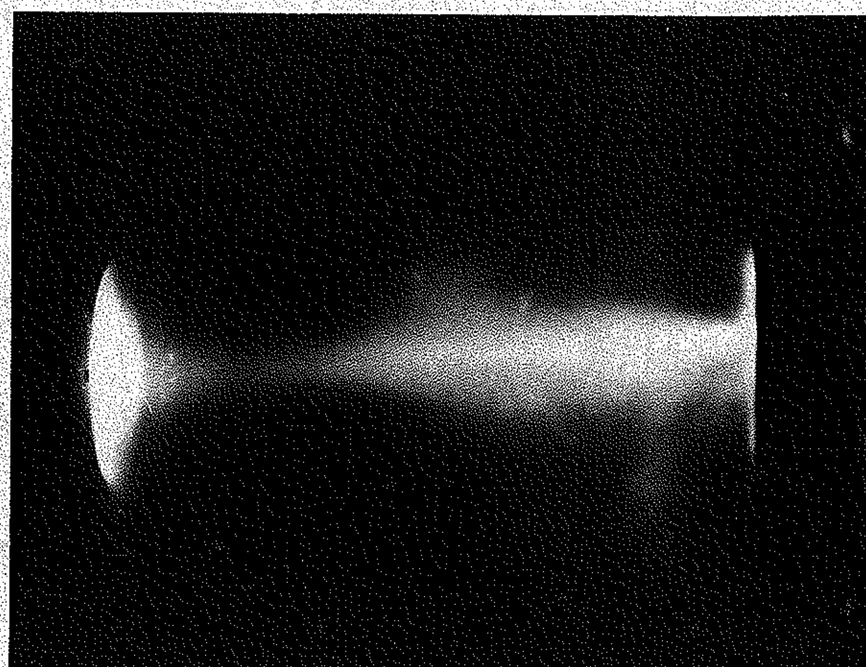


FIGURE 2b. Beam in air at .1 mm with calorimeter connected to chamber walls

pinched mode. If electrical breakdown occurs in a time much less than the beam rise time, then almost all of the return current can flow in the gas and there will be no pinch.

The breakdown explanation of electron beam behaviour is the subject of the remainder of this report. The rate of ionization build-up in the gas is considered in Sections 2 and 3. In Section 4 the observed behaviour of the electron beam is compared with predictions from the breakdown theory. There is rough order of magnitude agreement in the following areas:

1. The pressure dependence of the beam current density for air and helium,

2. The relation between electron kinetic energy and the maximum gas pressure at which a pinched beam is observed.

II. ELECTRICAL BREAKDOWN IN GASES

When the electron beam first enters the drift chamber, the conductivity of the gas is very low. The electron beam creates a highly ionized plasma. The time required for the degree of ionization to build-up to a highly conducting level can have a major influence on the amount of pinching which will occur.

We will consider first the ordinary breakdown process which occurs when an electric field is applied to an unionized gas. In general, the electrical breakdown of a gas is due to the build-up of ionization from processes occuring in the gas and at the electrodes and walls of the discharge chamber. For certain conditions the electrode and wall effects may be neglected. The results of breakdown experiments using microwave frequencies (Ref. 4) and nanosecond pulses (Ref. 5) are in good agreement with theoretical models which neglect electrode and wall effects.

The expression for the rate of change of the electron density is (Ref. 5)

$$\frac{dn}{dt} = \nu_i n - \nu_a n - \nabla \cdot \vec{\Gamma} \qquad (1)$$

where n is the electron density

ν_i and ν_a are the ionization and attachment frequencies

Γ is the particle flow

In terms of the electron drift velocity, v

$$\Gamma = nv$$

The recombination loss of electrons at electron densities in the neighborhood of breakdown is negligible and has been neglected.

If the electrode and wall effects can be neglected and the electric field is uniform one can assume that

$$\nabla \cdot \vec{\Gamma} = 0$$

The range of parameter for which this assumption is valid as well as other limits for this theory have been evaluated for plane parallel electrodes in air (Ref. 5) in terms of four limit lines (see Figure 3). The parameters are P, gas pressure, τ, breakdown time, and $\Lambda = d/\pi$ where d is the plate separation.

The mobility amplitude limit is determined by equating the electron drift distance during the breakdown time with the plate separation. The uniform field limit is a requirement that the breakdown time cannot be shorter than the time required to propagate the voltage pulse across the gap. The mean free path limit occurs when the mean free path is equal to the gap separation. The theory requires many collisions during the formative time. For this reason, the collision frequency limit is determined by equating the breakdown time and the mean collision time.

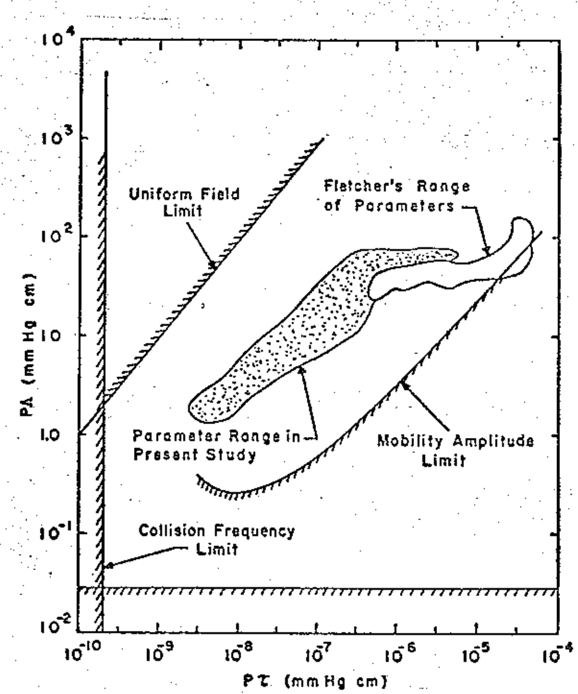


FIGURE 3. Validity-limit lines for air

III. IONIZATION CAUSED BY THE COMBINED EFFECTS OF THE ELECTRIC FIELDS OF THE BEAM AND COLLISIONS WITH BEAM ELECTRONS

In addition to the ions created by the interaction of the electric field of the beam and the background gas, ions are formed by collisions between the beam electrons and the gas atoms. The rate of ion production by the second process is:

$$\left(\frac{dn}{dt}\right)_{col} = \frac{j}{q} N \sigma_h$$

where j is the current density in the beam

q is the charge of the electron

σ_h is the ionization cross section for the relativistic beam electron

The differential equation describing the rate of change of the secondary electron density in the gas due to both processes is

$$\frac{dn}{dt} = (\nu_i - \nu_a)\, n + K\, j + \nabla \cdot \vec{\Gamma} \qquad (2)$$

$$K = \frac{N \sigma_h}{q}$$

An exact description of the ionization build-up in the background gas is an exceedingly difficult problem. In general $\nabla \cdot \vec{\Gamma} \neq 0$. ν_i and ν_a are functions of the applied electric field. During the rise time of the beam, pulse the current density, j, may be changing in a complicated fashion because the total beam current increases continually and additionally the beam may be pinching at the same time. The electric fields associated with the beam are not uniform. The magnetic field of the beam will be changing due to the change of the current as well as the beam diameter; and this will effect the electron mobility in the gas.

In order to get some idea about the order of magnitude of the time required for electrical breakdown in the gas, Equation 2 will be solved for the following simplified case;

1. $\nabla \cdot \vec{\Gamma} = 0$
2. The electric field is uniform and constant
3. The effect of the magnetic field can be neglected,
4. The current density during the rise time of the beam pulse, t_r is given by

$$j = j_o \frac{t}{t_r}$$

where j_o = current density during the flat top of the beam pulse.

For these conditions Equation 2 reduces to

$$\frac{dn}{dt} = (\nu_i - \nu_a)\ n + \frac{K\, j_o}{t_r}\ t \qquad (3)$$

in which ν_i, ν_a and $\frac{K j_o}{t_r}$ do not depend on t.

The solution of Equation 3 is

$$n = n_o e^{t/t_i} + \frac{K\ j_o}{t_r (\nu_i - \nu_a)^2} \left[e^{t/t_i} - \frac{t}{t_i} - 1 \right] \qquad (4)$$

where n is the electron density (electrons/cm^3)

n_o is the electron density for t = o (electrons/cm^3)

ν_i and ν_a are the ionization and attachment frequencys (sec^{-1})

$t_i = (\nu_i - \nu_a)^{-1}$ = mean ionization time (sec)

N is the gas particle density (atoms/cm^3)

σ_h is the cross section for ionization by the relativistic beam electrons (cm^2)

q is the electronic charge (coulombs)

K is $\frac{N \sigma_h}{q}$

j_o is the current density in the beam during flat top (amps/cm^2)

t_r is the beam rise time (sec)

The mean ionization time t_i can be evaluated from published data for the electron drift velocity (v), the Townsend coefficient (α), and the attachment coefficient (β)

$$t_i = \frac{1}{\nu_i - \nu_a} = \frac{1}{v(\alpha - \beta)}$$

v, $\frac{\alpha}{p}$ and $\frac{\beta}{p}$ are functions of the ratio of electric field, E, to gas pressure, p, so that a useful expression is

$$pt_i = \frac{1}{v\left[\frac{\alpha}{p} - \frac{\beta}{p}\right]}$$

pt_i is then a function of $\frac{E}{p}$

A compilation of the best available experimental data for v, $\frac{\alpha}{p}$, and $\frac{\beta}{p}$ was used in Reference 5 to predict values of breakdown times (in the nanosecond region) for a spark gap with a plane parallel geometry. The agreement with measured values of breakdown time was quite good.

Figures 4 and 5 are plots of pt_i versus E/p from the data in Reference 5. Figures 6 and 7 are plots of t_i versus pressure for constant values of field strength, E.

The constant, K, can be evaluated from data on the rate of energy loss by ionization for relativistic electron passing through a gas. For a temperature of $0^\circ C$

$$K = \frac{1}{q} \frac{p}{760} \frac{1}{\Delta T_o} \frac{dT}{dX}$$

where ΔT_o is the average energy lost per ion par (ev)

$\frac{dT}{dX}$ is the rate of energy loss by ionization at a pressure of 760 mm Hg (ev/cm)

p is the gas pressure (mm Hg)

q is the electronic charge (coulombs).

Use has been made of the fact that the average energy lost per ion pair is 32.5 ev for air and 42.3 ev for helium (Ref. 6). The variation of $\frac{dT}{dX}$ with electron energy in the range 2.5 to 10 MeV is not great and an average value of 2.30×10^3 ev/cm for air and 3.5×10^2 ev/cm for helium (Ref. 7) has been used to give the relations (0^oC)

$$K_A = 5.82 \times 10^{17} p \quad \text{for air}$$
$$K_{He} = 8.83 \times 10^{16} p \quad \text{for helium} \qquad (5)$$

The value of the initial electron density, n_o, in Equation 4 is highly uncertain, but it is almost certainly small compared to the quantity

$$\frac{K \; j_o}{t_r (\nu_i - \nu_a)^2}$$

so that from Equations 4 and 5 the electron density is given approximately by the expression

$$n = 5.82 \times 10^{17} \; p \frac{j_o}{t_r} \; t_i^2 \; [e^{t/t_i} - \frac{t}{t_i} - 1] \quad \text{for air}$$
$$n = 8.83 \times 10^{16} \; p \frac{j_o}{t_r} \; t_i^2 \; [e^{t/t_i} - \frac{t}{t_i} - 1] \quad \text{for helium} \qquad (6)$$

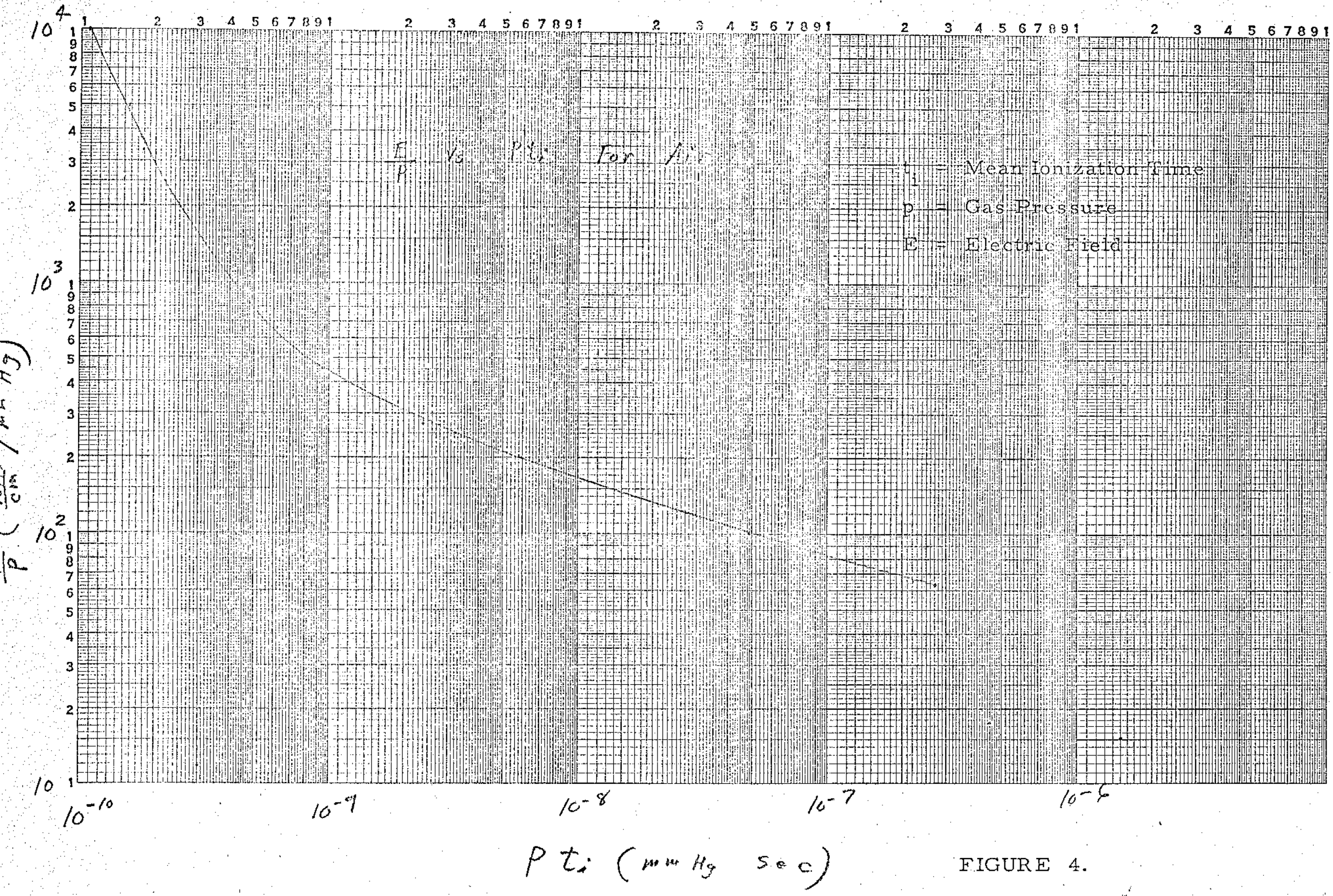

E/p vs Pti For Air
t_i = Mean Ionization Time
p = Gas Pressure
E = Electric Field
E/p (v/cm / mm Hg)
P ti (mm Hg sec)
10^4
10^3
10^2
10
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}

FIGURE 4.

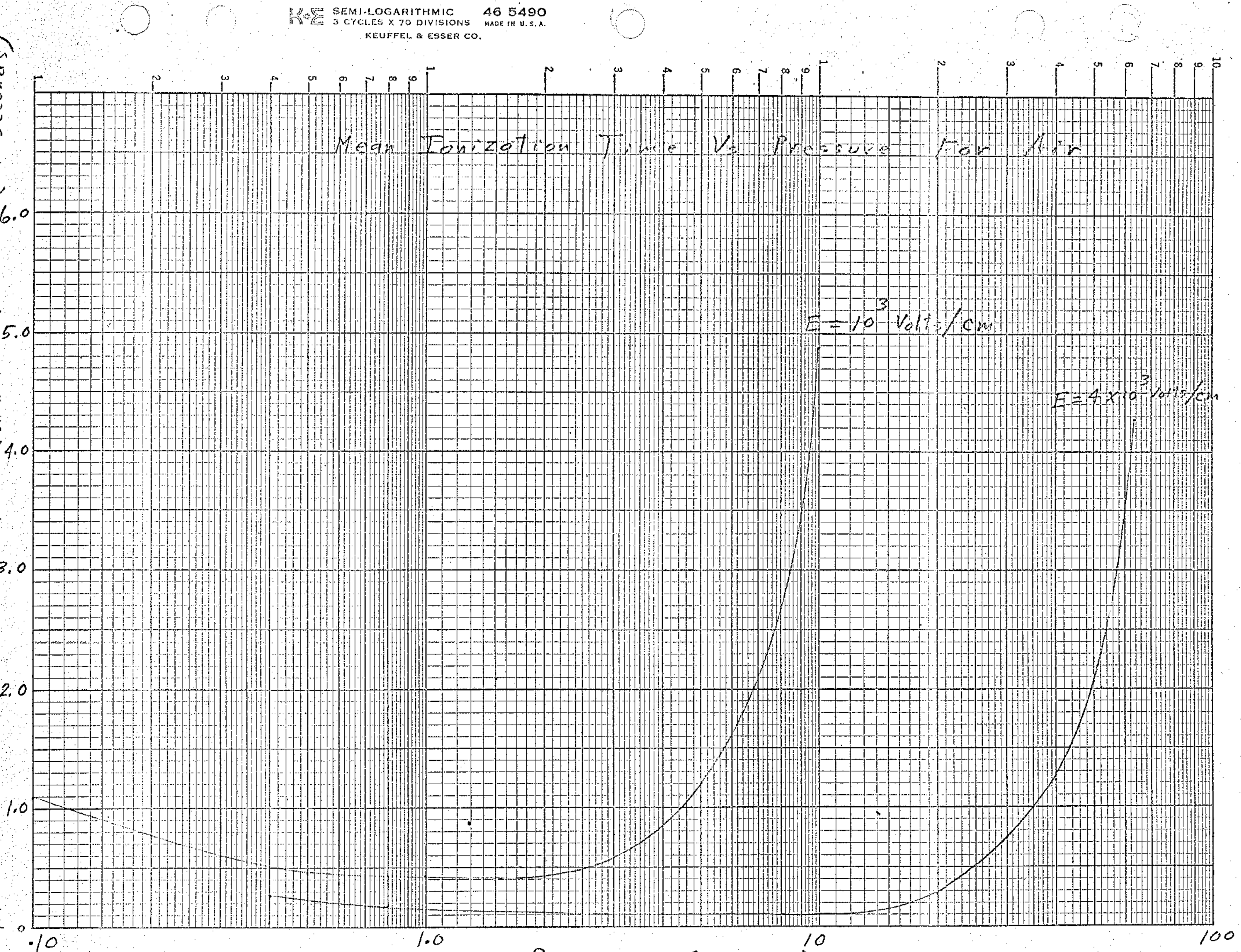

K+E SEMI-LOGARITHMIC 3 CYCLES X 70 DIVISIONS KEUFFEL & ESSER CO. 46 5490 MADE IN U.S.A.
Mean Ionization Time Vs Pressure For Air
E = 10^3 Volts/cm
E = 4 x 10^3 Volts/cm
6.0
5.0
4.0
3.0
2.0
1.0
0
.10
1.0
10
100
Pressure

FIGURE 6

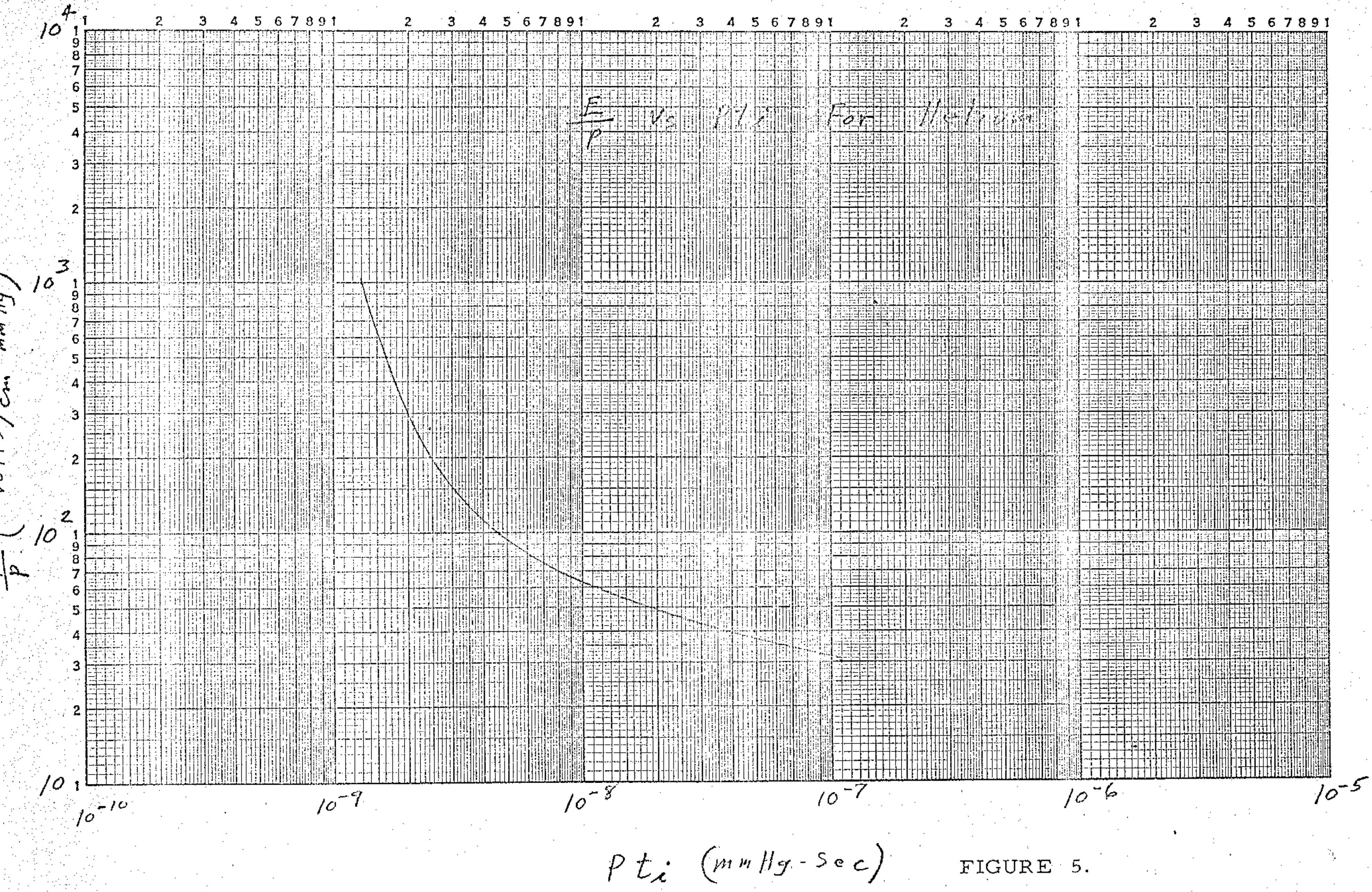
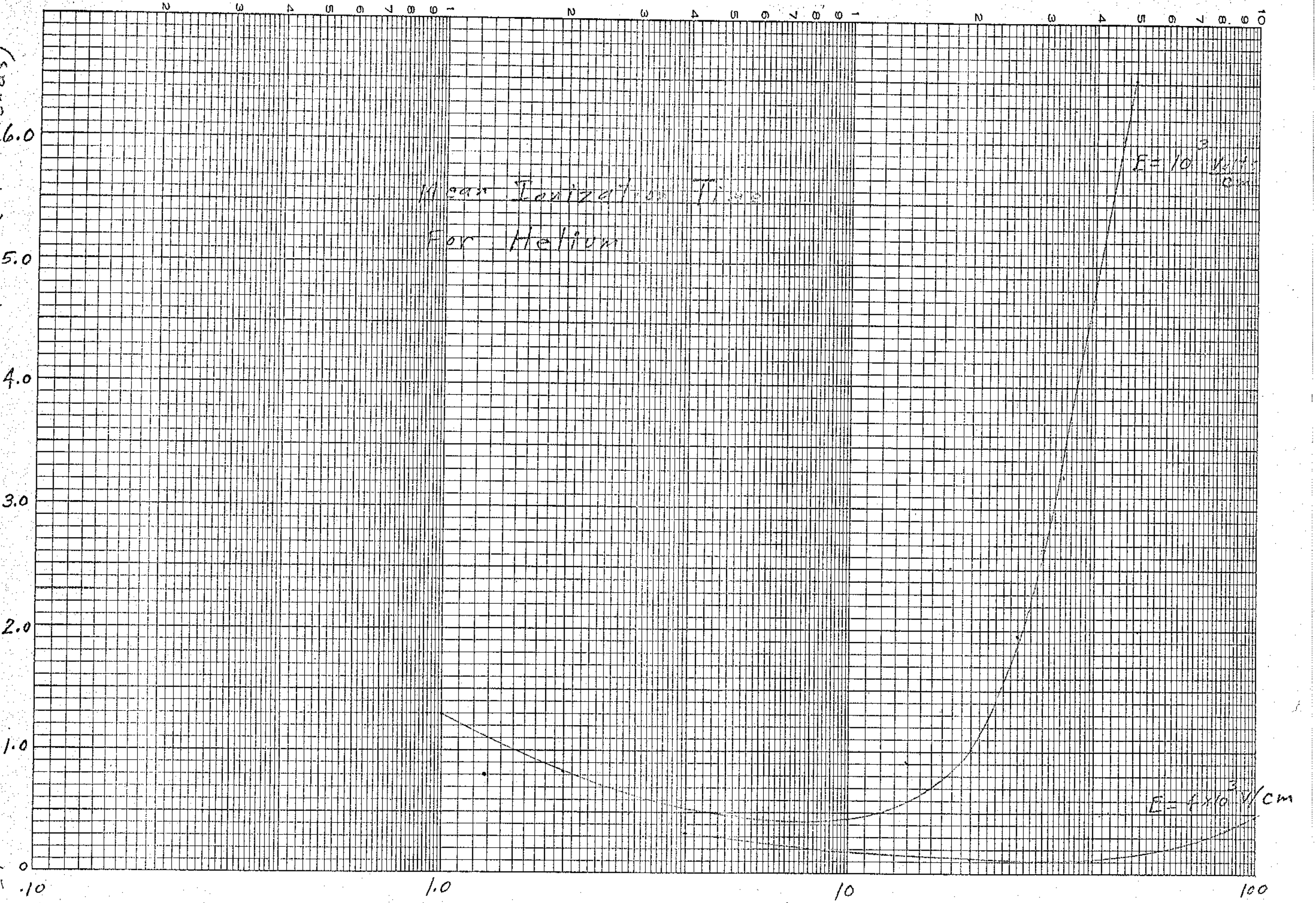
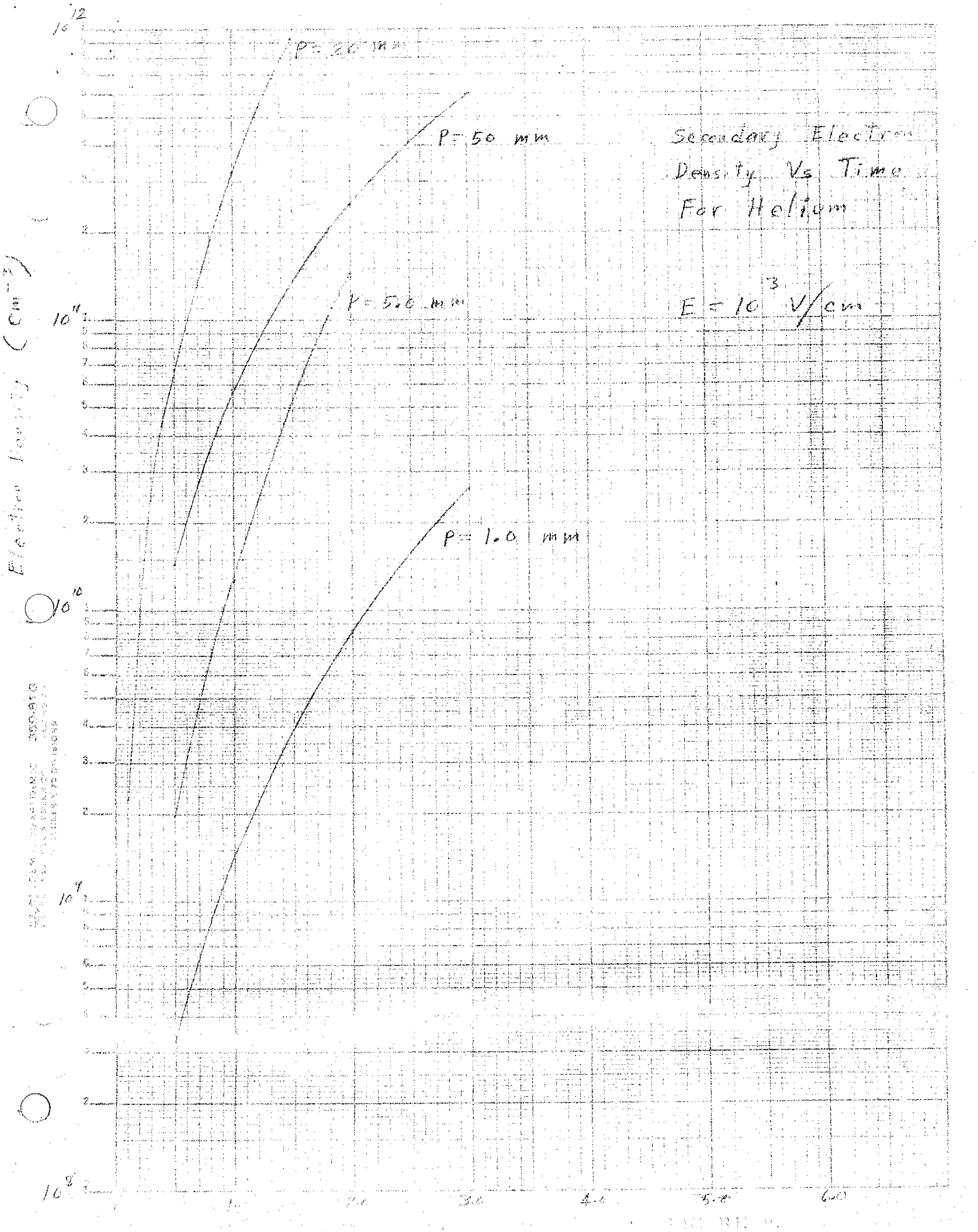

E/P vs Pti For Helium
10^{4}
10^{3}
10^{2}
10
10^{-10}
10^{-9}
10^{-8}
10^{-7}
10^{-6}
10^{-5}
Pti (mm Hg - Sec)

FIGURE 5.

NO. 340-L310 DIETZGEN GRAPH PAPER
SEMI-LOGARITHMIC
3 CYCLES X 10 DIVISIONS PER INCH

EUGENE DIETZGEN CO.
MADE IN U. S. A.





Secondary Electron Density Vs Time For Helium
E = 10³ V/cm
P = 20 mm
P = 50 mm
P = 5.0 mm
P = 1.0 mm
Electron Density (cm⁻³)
10¹²
10¹¹
10¹⁰
10⁹
10⁸
1.0
2.0
3.0
4.0
5.0
6.0

These expressions are plotted in Figures 8 and 9 for the following parameters:

$$E = 10^3 \text{ V/cm}$$

$$\frac{dI}{dt} = 5 \times 10^{11} \text{ Amps/sec}$$

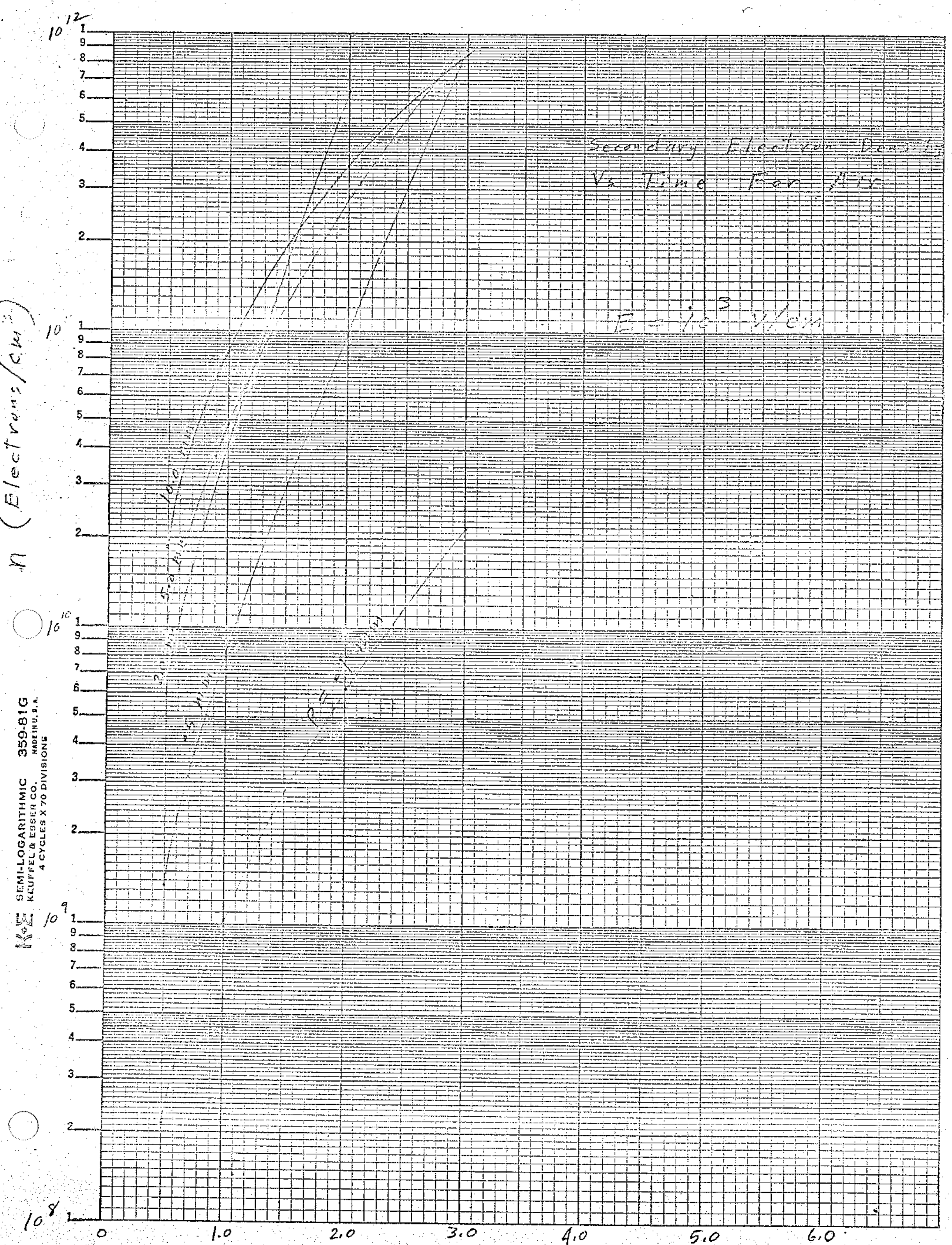
$$\frac{dj}{dt} \cong \frac{j_o}{t_r} = 2.5 \times 10^{10} \frac{\text{Amps}}{\text{cm}^2 \text{ sec}}$$

The criteria for electrical breakdown used in Reference 4 and 5 was that breakdown occured when the ratio of electron density to initial electron density (n/n_o) was 10^8. The value of the breakdown time is insensitive to the exact value of this ratio because of the exponential nature of the build-up process, and the value of 10^8 was found to give good agreement with measured data for both microwave and nanosecond breakdown experiments.

The precise value of the initial electron density (n_o) is uncertain. This uncertainty introduces no difficulty in the normal breakdown calculations (i.e. if the second term on the right side of Equation 4 is zero) because the solution involves the ratio n/n_o. However, when the effects of collisions with beam electron are included, it is not possible to apply the breakdown criteria $(n/n_o = 10^8)$ of Reference 4 and 5 directly.

If the distribution of currents and electric fields in the drift chamber are known it would be possible to treat the breakdown criteria as a parameter to be determined by experiment. Lacking this information one could make a crude estimate in the following way. The steady state conductivity of a weakly ionized gas is given by (Ref. 8).

$$\sigma = .532 \frac{\eta q^2}{(mkT)^{1/2}} \quad \frac{1}{Q} \tag{7}$$



Secondary Electron Density
Vs Time For Air
E = 10^3 V/cm
n (Electrons/cm^3)
10^12
10^10
10^9
10^8
0
1.0
2.0
3.0
4.0
5.0
6.0
SEMI-LOGARITHMIC 359-81G
KEUFFEL & ESSER CO. MADE IN U.S.A.
4 CYCLES X 70 DIVISIONS

FIGURE 8

where η is the ratio of electron density to gas atom density

q and m are the electron charge and mass

T is the electron temperature

Q is the electron atom collision cross section averages over the electron speed distribution.

The quantities in Equation 7 are in ESU and the conductivity should be divided by 9×10^{11} to give values in mho/cm. For an electron density of 10^9 electron/cm^3 and a gas density of 0.1 mm Hg the resistivity from Equation 7 is 10-100 ohm cm. One would expect the resistivity of a completely ionized gas to be around 10^{-2} ohm cm. It does not seem unreasonable to assume that electrical breakdown corresponds to an electron density on the order of 10^9 electrons/cm^3 or greater.

From Figure 8 one can see that the time required to reach any given electron density, greater than 10^9 electrons/cm^3, is about twice as long for a pressure of 0.1 mm as for a pressure of 0.5 mm.

IV. COMPARISON OF THE BREAKDOWN THEORY AND THE OBSERVED BEHAVIOR OF THE ELECTRON BEAM.

In order to compare predictions from the preceeding theory with experimental data, it is necessary to have some estimate of the electric fields in the drift chamber, as well as the rate of change of the beam current and current density. Since the transit time of light across the drift chamber is small compared to the rise time of the beam pulse, the approximate magnitude of the electric field can be calculated for the quasi-static case.

$$\vec{E} = -\nabla\Phi - \frac{\partial\vec{A}}{\partial t} = \vec{E}_c + \vec{E}_m$$

where the retardation effects are neglected in calculating Φ and $\vec{A}$ from the charges and currents. Since only a rough estimate can be hoped for at best, the electric field will be calculated for a non diverging uniform beam of radius r_1 located coaxially in a drift chamber of radius r_2 and length $\ell >> r_2$.

For this case the electric field at the center of the chamber due to the charge in radial and is given by (Mks units)

$$E_c = \frac{\rho r_1^2}{2 r \varepsilon_o} \qquad r_1 \le r \le r_2$$

$$E_c = \frac{\rho r}{2\varepsilon_o} \qquad o \le r \le r_1 \qquad (8)$$

where ρ is charge density and

ε_o is 8.85×10^{-12}

The electric field due to the changing magnetic field is longitudinal and is given by (neglecting the displacement current)

$$E_m = \frac{\mu_o}{2\pi} \frac{dI}{dt} \left[\frac{r_1^2 - r^2}{2 r_1^2} + \ell n \frac{r_2}{r_1} \right] \qquad o \le r \le r_1$$

$$E_m = \frac{\mu_o}{2\pi} \frac{dI}{dt} \ell n \frac{r_2}{r} \qquad r_1 \le r \le r_2 \qquad (9)$$

For a current density of the form

$$j = j_o \frac{t}{t_r}.$$

the electric field due to the space charge of the beam electrons only would be ($\beta = 1$)

$$E_c = \frac{j_o r}{2\varepsilon_o c} \frac{t}{t_r} = 188\, j_o r \frac{t}{t_r} \qquad o \le r \le r_1$$

for $j_o = 7.5 \times 10^2$ A/cm^2

$r = 1$ cm

$t = 1$ ns

$t_r = 10$ ns

$$E_c = 1.41 \times 10^4 \text{ V/cm}$$

However, from Figure 8 one can see that for t = 1 ns the build-up of positive ion density is appreciable.

For example, if p = .5 mm, t = 1 ns and E = 10^3 V/cm, the charge density of the positive ions would be about half that of the electron beam charge density. For an electric field of 10^4 V/cm the charge density of the positive ions would be much higher than the electron beam charge density. If the secondary electrons can escape from the beam, it seems reasonable to assume that the electric field due to space charge is self limiting; and that the electric field due to the changing magnetic field (which is unaffected by ionization build-up before breakdown) dominates the secondary processes in the gas.

For the electron beam shown in Figure 1, the rise time of the bremsstrahlung pulse is about 10^{-8} seconds and the current rise time is presumably somewhat longer. The maximum current is about 2 x 10^4 amps giving a value of $\frac{dI}{dt}$ around 10^{12} amps per second during the rise time. For this value of $\frac{dI}{dt}$ and reasonable value of r_2 and r_1 the electric field (Em) from Equation 9 is 10^3 to 10^4 volts/cm near the center of the beam.

Figure 10 is a plot of measured values of beam energy density versus drift chamber pressure from data in Reference 9. Comparing the data in Figure 10 for air with the curves in Figure 8 (E = 10^3 V/cm) it can be seen that the fraction of the beam rise time required to reach an electron density of 10^9 electrons/cm^3 or greater drops sharply with pressure in the 0.1 to 2 mm range and there is a corresponding drop in beam intensity in Figure 10. Furthermore, the optically observed pinch (see Figure 1) ceases to exist as the pressure is increased in the 0.1 to 0.35 mm region and there is a particularily sharp reduction in the ionization build-up time for this pressure range.

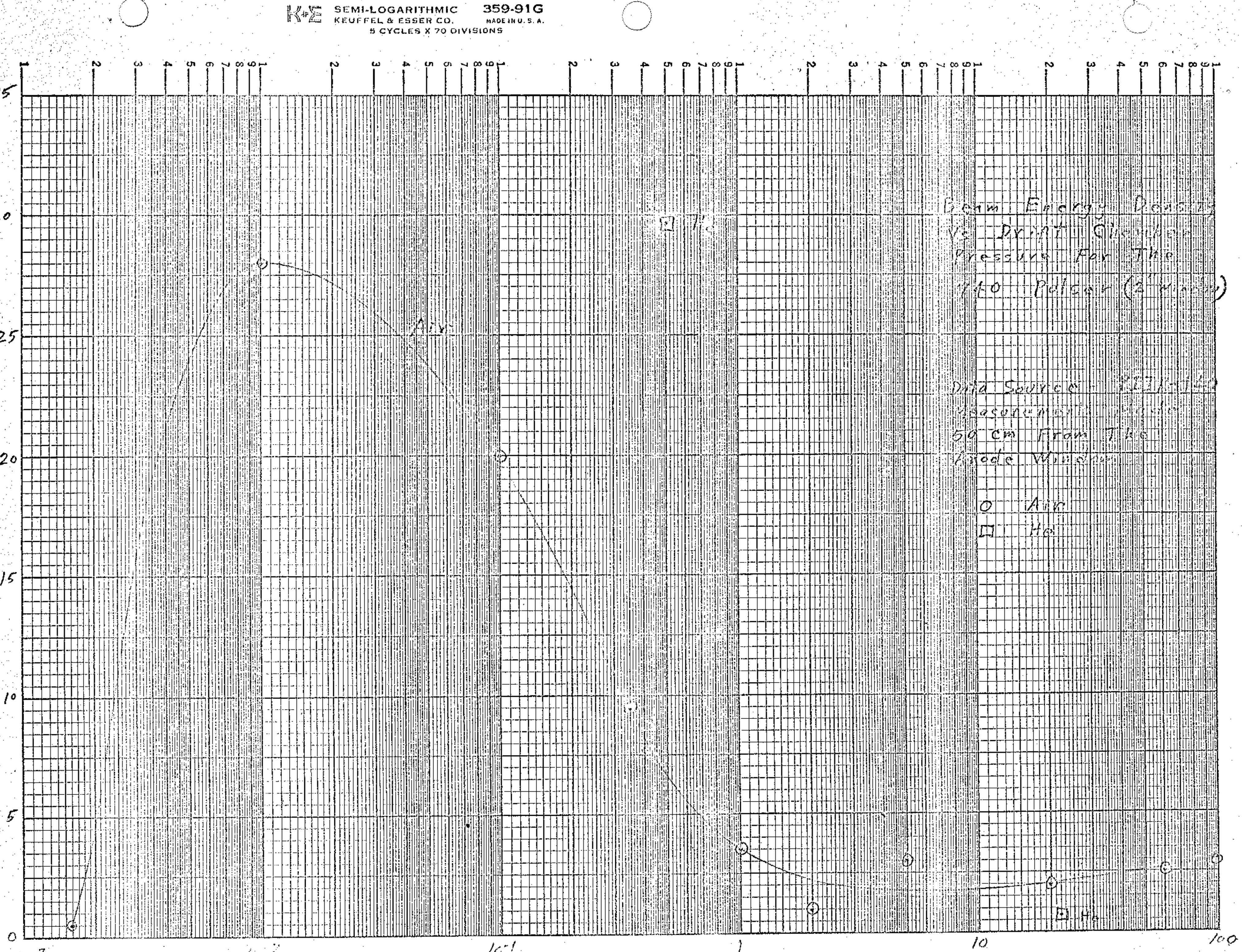

SEMI-LOGARITHMIC 359-91G
KEUFFEL & ESSER CO. MADE IN U.S.A.
5 CYCLES X 70 DIVISIONS
Air
O Air
□ He
□ He
35
30
25
20
15
10
5
0
10⁻³
10⁻¹
1
10
100

FIGURE 10

Unfortunately the experimental data for helium consists of only two points. From Figures 6, 7, 8 and 9 one can see that the behavior of helium is very similar to that of air except that the curve of mean ionization time versus pressure is shifted upward in pressure, and there is a corresponding shift in pressure for the curves of ionization build-up versus time (Figure 9). The two data points for helium in Figure 10 are in general agreement with this pressure shift. Pictures of the electron beam in helium at 0.5 mm display the pinched mode of operation.

Table I lists the maximum pressures for the pinched mode of operation in air, for electron beams having several values of electron kinetic energy.

There is a definite decrease in the maximum pressure for pinching as the electron kinetic energy is increased. The phenomena can be explained on the basis of the breakdown theory in the following way. As the kinetic energy of the electrons is increased, larger magnetic fields are required to form a pinch. Since $\frac{dI}{dt}$ is approximately the same for all of the beams in Table I, larger magnetic fields are produced by lowering the pressure and causing breakdown to occur latter (i.e. at higher values of current).

V. CONCLUSION

The time required for ionization build-up in the background gas is the right order of magnitude to explain the pressure dependence of the pinched mode for the electron beam. The pressure dependence of the rate of ionization build-up is considerably different for helium than for air, and the observed behavior is in qualitative agreement with this result.

Much work on the problem remains to be done. In particular, the ionization process should be investigated for electric fields which change with position and time. The distribution of secondary electrons and the electric fields should be investigated more throughly. The pressure dependence of the beam intensity should be measured in helium. The time history of the light from the beam path should be measured for air and helium as a function of pressure.

Machine	Shot No.	Cathode Anode Spacing (cm)	Max. Pinch Pressure (mm Hg)	Rise Time of Photo Diode Pulse (nanosec)	Pulse Width (FWHM of Photo Diode Pulse (nanosec)	Approx. Kinetic Energy (MeV)	Approx. Current (Amps)
310	669, 679	1	.5 to 1	~5*	~25*	.9	2.3×10^4
310	656	2	≤ .5	~5*	~25*	1.2	-
730 (B[2])	4715, 4739	6.8, 7	.1 to .35	7, -	34, 32	3	2.3×10^4 (2" window)
1148 (B[3])	1368, 1280	14	.06 to .2	50, 30	40, 40	4.3	8.2×10^4

TABLE I

*Estimate Made From Other Shots

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